MA 335, Spring 2014

Assignment 3

This homework is due Friday Feb 7.

There are total 43 points in this assignment. 39 points is considered 100%. If you go over 39 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge.* Your solutions should contain full proofs. Bare answers will not earn you much.

Please note that in this assignment you are asked to solve problems using Linear expression of gcd and Euclidean algorithm (Chapter 2 material in textbook), but not unique prime factorization (Chapter 3).

(1) (2.3.14+) For any integer a, show the following:

  (a) [1pt] $\gcd(a, 5a+1) = 1$.
  (b) [2pt] $\gcd(2a+1, 9a+4) = 1$.
  (c) [2pt] $\gcd(5a+2, 7a+3) = 1$.

(2) [3pt] If $\gcd(a, b) = 1$, show that $\gcd(2a+b, a+2b) = 1$ or 3.

(3) [3pt] Show that $\gcd(a, b)$ divides $\gcd(a+b, a-b)$. Is it true that always $\gcd(a, b) = \gcd(a+b, a-b)$? (Prove or provide a counter example.)

(4) [2pt] (2.4.1) Use Euclidean algorithm to find $\gcd(143, 227)$, $\gcd(272, 1479)$.

(5) (2.4.2bc) Use the reverse Euclidean algorithm to obtain integers x, y such that:
  (a) [3pt] $\gcd(24, 138) = 24x + 138y$.
  (b) [3pt] $\gcd(119, 272) = 119x + 272y$.

(6) (a) [2pt] (2.3.20(a)) Deduce directly from Linear expression of gcd that if $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$, then $\gcd(a, bc) = 1$. (*Hint:* Write $1 = 1 \cdot 1 = (ax+by)(au+cv)$. Expand brackets, combine terms with a and terms with bc.)
  (b) [2pt] Use item (a) to prove that if $\gcd(a, b) = 1$, then $\gcd(a, b^n) = 1$ for all integer $n \geq 1$.
  (c) [2pt] Use items (a) and (b) to prove that if $\gcd(a, b) = 1$, then $\gcd(a^n, b^m) = 1$ for all integer $m, n \geq 1$.

(7) Determine which of the following Diophantine equations can be solved:

  (a) [2pt] (2.5.1c) $14x + 35y = 93$,
  (b) [2pt] (2.5.1b) $33x + 14y = 115$,
  (c) [2pt] $33x + 57y - 9z = 10$.

(8) (2.5.2ac) Find all solutions in the integers of the equation (use reverse Euclidean algorithm)
  (a) [4pt] $56x + 72y = 40$.
  (b) [4pt] $221x + 35y = 11$.

(9) [4pt] (2.5.3b) Determine all solutions in the *positive* integers of the equation (use reverse Euclidean algorithm) $54x + 21y = 906$.